INSTABILITY AND MULTISTABILITY IN RAPID GRANULAR SHEAR FLOWS

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It has been recently discovered that an important generic feature of rapidly deforming (granular) systems of inelastically colliding particles is their tendency to form dense particle clusters of low kinetic energy within dilute ambients of energetic particles. The thesis of the present paper is that for systems subject to external shearing the mechanism responsible for the formation of clusters is a nonlinear instability involving the combined action of long-wavelength hydrodynamic modes, convective transport (i.e. stretching by the shear) and local inelastic dissipation. The time scale for cluster formation is found to be inversely proportional to the average fluctuating kinetic energy ('granular temperature') of the particles in the system. The degree of clustering in a system depends significantly on the rate of this process relative to those of other competing processes such as diffusion and convection. As a result, a sheared system in which the granular temperature is relatively high (e.g. one prepared in an initial state whose granular temperature is much higher than that in the statistically steady state to which the system evolves) exhibits a markedly different dynamics from one in which the temperature is low, all other externally imposed parameters for the two systems being the same. This is one of the sources of hysterectic behavior in granular systems: it leads to the existence of multiple steady states which correspond to the same external parameters yet are characterized by very different microstructures. A model for these multistable states is presented and results of numerical simulations are used to demonstrate the variance of the steady state into which a system may evolve with respect to the initial conditions and the application of transient forces. Other possible mechanisms for multistability are briefly discussed.

1 Introduction

The notion of a 'granular fluid' arises when a system of macroscopic grains, such as sand or coal particles, are subject to such rapid deformation rates that the contacts between individual grains do not endure and their motion is rapidly randomized by frequent collisions. The microscopic dynamics of such a system bears an obvious analogy to that of a regular fluid, the main difference being that the collisions in the granular fluid are inelastic. Despite this analogy, granular fluids are rheologically very different from classical fluids. A most significant difference is the tendency of granular systems to form dense clusters of particles of low kinetic energy within dilute ambients of energetic particles. These clusters and other related 'inelastic microstructures' have been observed numerically in in chute flows[1], shear flows[2, 3], and in free flows[4, 5] (i.e. in systems that are left to decay from initial energetic states). Shear flows exhibit hysteresis: different flow states may arise depending on the history of the system, and the microstructures corresponding to these states can be significantly different from one another. Other unusual dynamical effects, such as normal stress differences[6], inelastic collapse[7, 8], generalized phase transitions[5], and oscillations[9, 10] are also found in granular fluids; some of these are reviewed in [11, 12].

The abovementioned analogy to the kinetics of classical fluids led to derivations of constitutive relations that parallel similar derivations in the classical kinetic theory of gases. The output of these derivations (cf. [13] and references therein) is a set of continuum equations for the kinetic energy density ('granular temperature') and for the mass and momentum densities, which have been moderately successful in explaining results of experiments and numerical simulations. However, these equations still fall short in some respects, most of which have to do with the fact that the existence of microstructures (i.e. strong inhomogeneities) has not been taken into account in their derivation. It seems, though, that these equations can successfully be used for the analysis of the dynamics of microstuctures in nearly elastic and nearly homogeneous systems. A stability analysis of these equations for the case of a free (unforced) system reveals that while homogeneous solutions do exist they are unstable to infinitesimal (inhomogeneous) perturbations[14, 5]. Also, a nonlinear instability was shown to be responsible for the clustering process in this case[4, 5]. Shear flows in both two- and three-dimensions, unlike unforced granular systems, are linearly stable, though some eigenmodes may grow for a finite time before reverting to temporal decay. This transient instability has been found to exist in homogeneous states in which the velocity profile is linear[15, 16, 18], although it is expected that a similar phenomenon exists in more complex setups as well. The stability analyses show that the orientation of the wavevector that correspond to the (transiently) most unstable mode coincides approximately with the extensional axis of the shear, which corresponds to disturbances that lie in real space at 135 degrees from the streamwise direction.

The rest of the paper is organized as follows: Section 2 presents a linear stability analysis of simple shear flows. In Section 3 the various time scales characterizing the dynamics of fluctuations in the shear flow are presented and the nature of the nonlinear mechanism which is responsible for the clustering process is elucidated. Particular emphasis is laid on the dependence of the typical time scales on the initial conditions and the way their values determine the major dynamical mechanisms and the ultimate, long-time fate of the system. Some results of large-scale 2-dimensional simulations of shear flows, substantiating the theoretical model proposed, are also presented in Section 2 and Section 3. A brief summary and an account of other routes to multistability in shear flows are given in Section 4.

2 Linear Stability Analysis

A plot of the particle configuration in a typical simple granular shear flow is shown in Fig. 1. The plot is obtained from a simulation of the dynamics of a system of inelastic disks, whose collisions are characterized by a normal coefficient of restitution, in a square enclosure of dimensions $L \times L$ that is periodically extended in both the x- and y-directions. The periodic boundary conditions are applied in the Lagrangian frame corresponding to linear shear and are known as the 'Lees-Edwards' boundary conditions[19, 2, 3]. One may regard the above system as one whose horizontal boundaries, which are parallel to the streamwise direction, move at equal and opposite velocities $\pm U/2$, the velocity of the top boundary being positive. It is more accurate to think of the system as having a linear velocity profile in the streamwise direction x, such that the streamwise velocity $v_1 = \gamma y$ where $\gamma \equiv U/L$ is the shear rate.

The flow field depicted in Fig. 1 is statistically stationary. It contains relatively dense strips aligned along the extensional axis of the shear which are interspersed between relatively dilute strips of a similar orientation and size. The dense strips have a secondary inner structure consisting of dense clusters which are elongated along the strips in which they reside. The mechanism responsible for the emergence of clusters in this system can be elucidated by considering the linear stability of the equations derived in [17] for two dimensional flows of monodisperse disks from kinetic theory. In the low-density limit, which is the regime we are considering, these equations read:

$$\begin{aligned} \nu\left(\dot{T} + v_i\partial_i T\right) &= \frac{\sqrt{\pi}}{2}\sigma\partial_i\left(\sqrt{T}\partial_i T\right) - \nu T\partial_i v_i \\ &+ \frac{\sqrt{\pi}}{16}\sigma\sqrt{T}\,\mathrm{Tr}\hat{D}_{ij}^2 - \frac{8}{\sqrt{\pi}}\frac{\nu^2}{\sigma}\epsilon T^{3/2} \qquad (1) \\ \nu\left(\dot{v_j} + v_i\partial_i v_j\right) &= -\partial_j\left(\nu T\right) \\ &+ \frac{\sqrt{\pi}}{8}\sigma\left(\partial_i\sqrt{T}\right)\left(\partial_i v_j + \partial_j v_i - \delta_{ij}\partial_l v_l\right) \\ &+ \frac{\sqrt{\pi}}{8}\sigma\sqrt{T}\Delta v_j \qquad (2) \\ \dot{\nu} &= -\partial_i\left(\nu v_i\right) \qquad (3) \end{aligned}$$

where T is the granular temperature field defined as half the local average of the squared velocity fluctuations and ν is the volume (area) fraction of the particles (i.e. reduced density) which equals ρ/ρ_s where ρ is the density and ρ_s is the mass density of a solid particle; also σ is the diameter of a particle and ∂_i denotes $\partial/\partial r_i$, where $i = 1, 2$ indicates the cartesian components of the position vector $\mathbf{r}$. The summation convention for repeated indices is assumed. The coefficient of restitution, $\tilde{e}$, appears in $\epsilon \equiv 1 - \tilde{e}^2$, and $\mathrm{Tr}\hat{D}_{ij}^2$ is the viscous heating function given by

$$\mathrm{Tr}\hat{D}_{ij}^2 = 2\left[(\partial_j v_i)(\partial_i v_j) + (\partial_j v_i)^2 - (\partial_l v_l)^2\right]. \qquad (4)$$

Equations (1)-(3) admit a basic solution with constant volume fraction ν_0, temperature T_0 and a velocity field $\mathbf{v} = (\gamma y, 0)$. The value of T_0 is determined by the balance between the rate of energy pumping by viscous heating and the rate of energy loss by the inelastic collisions, corresponding respectively to the third and fourth terms on the r.h.s of (1), which yields:

$$T_0 = \frac{l^2\gamma^2}{\pi\epsilon} \qquad (5)$$

where $l \equiv \pi\sigma/8\nu_0$ is the mean free path corresponding to the basic state.

The linear stability of shear flows governed by equations derived from kinetic theory, which are similar to (1)-(3), has been investigated before[15, 16, 18]. In [15, 18], the linearized equations are analyzed by first transforming them to coordinates which travel with the local mean flow and then by performing a Fourier transform of the resulting equations. This procedure eliminates the coordinate-dependent convective terms in the original linearized equations, at the price of defining modes in terms of time dependent wavevectors, which are continually rotated by the mean shear. The resulting equations are not self-adjoint. Disturbances evolving from $t = 0$ are found to grow for short times (the growth rates depending on the nature of the variables used in the analysis, a point discussed in [14]) then decay. An elaborate analysis presented in [16] shows rigorously that simple shear flow is asymptotically linearly stable, though there can be transient growth of infinitesimal disturbances. Here we consider the transient evolution of infinitesimal disturbances of the basic state for times which are short enough for the effect of convection to be negligible. It is easy to see that the typical convective time scale is: $\tau_c \sim 1/\gamma$. For times much less than τ_c, fluid elements in the system may be considered to be unaffected by the mean flow and hence the convective terms in the equations may be dropped to a good degree of approximation.

In the following, it is convenient to nondimensionalize (1)-(3) by defining:

$$\tilde{\mathbf{r}} = \frac{\mathbf{r}}{L}, \quad \tilde{t} = \gamma t \tag{6}$$

$$\tilde{T} = \frac{T}{T_0}, \quad \tilde{\mathbf{v}} = \frac{\mathbf{v}}{U}, \quad \tilde{\nu} = \frac{\nu}{\nu_0} \tag{7}$$

It is also convenient to introduce the dimensionless quantity

$$\delta^* \equiv \frac{\ell}{L\sqrt{\epsilon}} \tag{8}$$

which, as shown below, corresponds to the ratio of the typical separation between clusters to the linear dimension of the system. The infinitesimal disturbances $\delta\tilde{T}$, $\delta\tilde{\nu}$, $\delta\tilde{\mathbf{v}}$ of the basic state are defined by:

$$\tilde{T} = 1 + \delta\tilde{T} \tag{9}$$

$$\tilde{\nu} = 1 + \delta\tilde{\nu} \tag{10}$$

$$\tilde{\mathbf{v}} = \tilde{y}\hat{\mathbf{x}} + \delta\tilde{\mathbf{v}}. \tag{11}$$

Assuming eigenmodes of the form $\exp(i\mathbf{k}\cdot\tilde{\mathbf{r}} + s\tilde{t})$, and substituting (6)-(11) in (1)-(3), dropping the convective terms and linearizing in the disturbances, one obtains:

$$s\begin{pmatrix} \delta v_1 \\ \delta v_2 \\ \delta\nu \\ \delta T \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{\epsilon}}{\pi}\delta^{*2}k^2 & 0 & -i\delta^{*2}k\cos\theta & ik\left(\frac{\sqrt{\epsilon}\delta^{*2}\sin\theta}{2\pi} - \delta^{*2}\cos\theta\right) \\ 0 & -\frac{\sqrt{\epsilon}}{\pi}\delta^{*2}k^2 & -i\delta^{*2}k\sin\theta & ik\left(\frac{\sqrt{\epsilon}\delta^{*2}\cos\theta}{2\pi} - \delta^{*2}\sin\theta\right) \\ -ik\cos\theta & -ik\sin\theta & 0 & 0 \\ ik(2\sqrt{\epsilon}\sin\theta - \cos\theta) & ik(2\sqrt{\epsilon}\cos\theta - \sin\theta) & -2\sqrt{\epsilon} & -\sqrt{\epsilon} - \frac{4}{\pi}\sqrt{\epsilon}\delta^{*2}k^2 \end{pmatrix}\begin{pmatrix} \delta v_1 \\ \delta v_2 \\ \delta\nu \\ \delta T \end{pmatrix} \tag{12}$$

where $\mathbf{k} = (k\cos\theta, k\sin\theta)$ and the tildes on the disturbances have been dropped for notational convenience.

Let $\bar{\delta} = \delta^* k$, which in dimensional units is equal to $(2\pi n l/L\sqrt{\epsilon})$ for the nth mode in $\mathbf{k}$-space, and consider the limit $\bar{\delta} < 1$. In practice, this is usually the limit of interest, since the dominant wavelengths L/n for relatively inelastic systems are found in the numerical simulations to be such that $\bar{\delta} < 1$. The characteristic equation corresponding to (12) is:

$$s^4 + \left(\sqrt{\epsilon} + \frac{6}{\pi}\bar{\delta}^2\sqrt{\epsilon}\right)s^3 + \bar{\delta}^2\left[2 - \sqrt{\epsilon}\left(2 + \frac{1}{2\pi}\right)\sin 2\theta + \frac{3}{\pi}\epsilon\right]s^2 + \bar{\delta}^2\left(\frac{\epsilon\sin 2\theta}{\pi} - \sqrt{\epsilon}\right)s + \mathcal{O}(\bar{\delta}^3) = 0 \tag{13}$$

where the terms of order $\bar{\delta}^3$ or higher have not been written out explicitly. The full solution of (13) (i.e. *including* the $\mathcal{O}(\bar{\delta}^3)$ terms) can be obtained numerically, and solutions for the largest positive root, s_m, for $\sqrt{\epsilon} = 0.8$ are shown in Fig. 2 and Fig. 3. These graphs show that for small values of $\bar{\delta}$, Re$[s_m]$ is largest when $\theta = 3\pi/4$. For larger values of $\bar{\delta}$, a cross-over occurs and Re$[s_m]$ becomes largest when $\theta = \pi/4$. The directions of maximal growth are interchanged again at still larger values of $\bar{\delta}$. A detailed numerical study of the solutions of (13) shows that Re$[s_m]$ is positive in the range $0 < \bar{\delta} < 1$ for all values of $\sqrt{\epsilon}$ less than approximately $1/\pi$. The directions of maximal growth are also found to remain in either the $\theta = \pi/4$ or $\theta = 3\pi/4$ directions. The real parts of the other roots of (13) are either negative or much smaller in magnitude than Re$[s_m]$.

For the case $\bar{\delta} \ll 1$, (13) allows for a perturbative study of its solutions through which the stability properties of the problem can be made more transparent. Neglecting the $\mathcal{O}(\bar{\delta}^3)$ terms, and noting that one root of (13) is $s_1 = 0$, we seek solutions for the other roots as series in δ and $\sqrt{\delta}$: $s_2 = \sum_{n=0}^{\infty} a_n\bar{\delta}^{2n}$ and $s_{3,4} = \sum_{n=1}^{\infty} b_n\bar{\delta}^n$. A substitution of the above ansatz into (13) yields the following solutions:

$$s_2 = -\sqrt{\epsilon} + \left[\frac{3}{\sqrt{\epsilon}} - \frac{3\sqrt{\epsilon}}{\pi} - \left(2 + \frac{3}{2\pi}\right)\sin 2\theta\right]\bar{\delta}^2 + \mathcal{O}(\bar{\delta}^4) \tag{14}$$

$$s_3 = \sqrt{1 - \frac{\sqrt{\epsilon}\sin 2\theta}{\pi}}\,\bar{\delta} + \mathcal{O}(\bar{\delta}^3) \tag{15}$$

$$s_4 = -\sqrt{1 - \frac{\sqrt{\epsilon}\sin 2\theta}{\pi}}\,\bar{\delta} + \mathcal{O}(\bar{\delta}^3) \tag{16}$$

For small values of $\bar{\delta}$, it is seen that the only growing mode corresponds to s_3 and that the maximal growth rate occurs in the $\theta = 3\pi/4$ direction. For long enough wavelengths, transient linear growth is largest at 45 degrees from the streamwise direction; for intermediate wavelengths, the growth rate is largest at 135 degrees from the streamwise direction (note that the direction of the wavevector of a mode is at right angles to the direction of the strips of equal phase in real space).

3 Nonlinear Mechanism and Multistability

We next outline a nonlinear mechanism which is initiated by the transiently growing linear modes and which we propose as the mechanism leading to cluster formation in sheared systems. A large enough granular system, hence one having a large number of degrees of freedom, experiences statistical fluctuations of every macroscopic physical quantity except those that are strictly conserved. Consider a sheared hard-disk fluid at an 'initial' temperature T_i. Such a system may have a shear fluctuation (with a finite probability) of the form $\delta\mathbf{v} = (0, v_0\sin kx)$, where k is consistent with periodic boundary conditions in finite domain. Since equipartition is expected to hold during early times before the dynamics of the system becomes dominated by clustering, the (typical) amplitude v_0 of such a fluctuation can be estimated by computing the energy stored in the velocity field corresponding to this fluctuation and comparing the result, order-of-magnitude-wise, to mT_i, which is the energy per degree of freedom (m is the mass of a particle). It is easy to show that $v_0 \sim \sqrt{T_i/N}$, where N is the total number of particles in the system, and that the typical magnitude of $h \equiv \partial\delta v_2/\partial x$ is $h \sim k\sqrt{T_i/N}$. Let the value of k correspond to the expected dominant length scale (i.e. the intercluster separation; see below). If we assume that the corresponding length scale, k^{-1}, is long enough so that diffusion effects can be neglected with respect to viscous heating and inelastic energy decay (an assumption to be justified a-posteriori) then we can approximate (1) by:

$$\nu\dot{T} = \frac{\sqrt{\pi}}{16}\sigma\sqrt{T}\,\mathrm{Tr}\hat{D}_{ij}^2 - \frac{8}{\sqrt{\pi}}\frac{\nu^2}{\sigma}\epsilon T^{3/2}. \qquad (17)$$

Note that we are still considering the dynamics of the system at early times when the effect of convection can be neglected. Consider firstly the case when the initial temperature T_i is so large that h is large compared to γ. This happens when T_i is much larger than T_0, the asymptotic value of the temperature given by (5) (the more precise statement of the condition is $T_i \gg \sqrt{N}T_0$). The viscous heating function in this case can be approximated as follows:

$$\mathrm{Tr}\hat{D}_{ij}^2 = 2(\gamma+h)^2 \simeq 2h^2\left(1+\frac{2\gamma}{h}\right) + \mathcal{O}\left(\frac{\gamma^2}{h^2}\right). \qquad (18)$$

Assuming that the density can be considered to be fixed in (17) and that the shear fluctuation is practically stationary with respect to the rate of decay of the temperature to its asymptotic value (assumptions again to be justified a-posteriori), it can be shown that the solution to (17) is:

$$T(t) = \frac{l^2h^2}{\pi\epsilon}A(t)\left[1+\frac{2\gamma}{h}\right] + \mathcal{O}(\frac{\gamma^2}{h^2}) \qquad (19)$$

where

$$A(t) = \left[\frac{1-\alpha\exp(-\sqrt{\epsilon}ht)}{1+\alpha\exp(-\sqrt{\epsilon}ht)}\right]^2 \qquad (20)$$

where $\alpha \equiv \left[1-\sqrt{T_i/(l^2h^2/\pi\epsilon)}\right] / \left[1+\sqrt{T_i/(l^2h^2/\pi\epsilon)}\right]$. The condition that diffusion is slow relative to the rate of saturation of $T(t)$ to its final value and that the shear fluctuation is quasi-stationary can be shown to be equivalent to:

$$kl < \sqrt{\epsilon} \qquad (21)$$

This result, derived in detail in [20] and in [5] for free systems, is physically plausible since diffusion is important only at large k, and the shear fluctuation, being of hydrodynamic scale, can decay only by a diffusive (i.e. viscous) mechanism. When $\gamma/h \ll 1$, the temperature rapidly saturates to the value dictated by the (local) velocity field corresponding to the shear fluctuation and a corresponding temperature gradient is formed in the system. As a result, a pressure gradient is established as well, its value being given by:

$$P = \rho_s\nu T \simeq \rho_s\nu\frac{l^2h^2}{\pi\epsilon}A(t). \qquad (22)$$

Since $l = \pi\sigma/8\nu$, we have $P \sim \rho_s\sigma^2h^2/\epsilon\nu$, i.e. the pressure that is established is inversely proportional to the density. Thus the pressure in dense regions is low relative to the pressure in dilute regions, causing mass to be transfered from the latter into the former. The excess of mass causes the dense regions to cool at a faster rate (since the collision rate there is increased), causing a further reduction in the values of the pressure in them and thus to a further mass flow into them. All in all, a small departure from a state of uniform density will generate an internal pressure difference that amplifies the departure, leading finally to the formation of high density clusters. For clustering to be possible, the mass must agglomerate at a faster rate than that of it being stretched apart by the mean shear, and fast enough to render diffusive processes inefficient. The time scale for mass motion leading to clustering can be estimated from the part of (2) which contains the pressure induced forces alone: $\rho\dot{\mathrm{v}} = -\nabla P$. Using (3), it follows that $\ddot{\rho} = \Delta P$, and the time scale, τ_m, for mass motion is easily seen to be

$$\tau_m \sim \frac{1}{k}\sqrt{\frac{\rho}{P}} \sim \frac{\sqrt{\epsilon}}{klh}. \qquad (23)$$

The condition that τ_m be shorter than the convective time scale, τ_c, is therefore: $(\gamma/h)\sqrt{\epsilon} < kl$. When this is combined with (21), we have

$$\sqrt{\epsilon}\frac{\gamma}{h} < kl < \sqrt{\epsilon} \qquad (24)$$

Notice that h depends on T_i and hence its value can be made large enough so that $\gamma/h < 1$. Thus the condition (24) on k can be easily satisfied. In fact (24) also encompasses the assumption in the linear analysis that the mean flow may be considered static on the time scale for linear growth, since from (15) we have $1/s_3 \sim \sqrt{\epsilon}/kl\gamma$ and $1/s_3 < 1/\gamma$ implies that $kl > \sqrt{\epsilon}$. Since τ_m is shorter the larger the value of k (it takes less time for mass to move a distance k^{-1} the shorter the distance), the fastest and dominant clustering process will occur at the largest allowed value of k. Thus the clustering process occurs on the scale determined by $kl \sim \sqrt{\epsilon}$. Since mass accumulates at the minima of h (where the temperature is lowest), this scale corresponds to the typical separation between clusters in the flow.

The case $\gamma/h \gg 1$ (i.e. $T_i < T_0$) can be analyzed in a similar way and the corresponding conditions on k are

$$kl > \sqrt{\frac{\gamma}{h}}\sqrt{\epsilon} \qquad (25)$$

$$kl < \sqrt{\epsilon} \qquad (26)$$

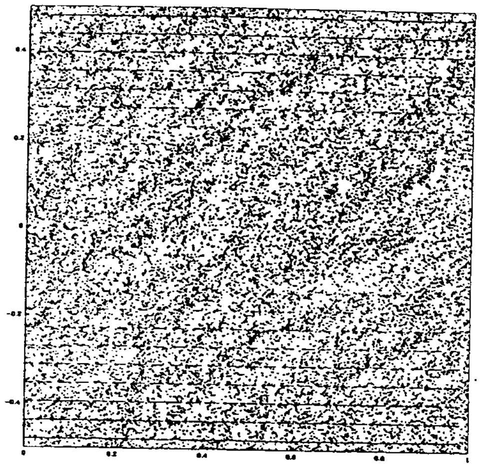

Figure 1: The particle configuration of a quasihomogeneous shear flow at steady state on which a vector plot of the velocity field is superposed. The parameters characterizing the flow are: $\tilde{e} = 0.6$, $\nu_0 = 0.05$, $N = 20000$.

Since (25) and (26) cannot be satisfied simultaneously, no clustering is possible at early times. Indeed, in this case, numerical simulations reveal growing modes in both the 45 and 135 degrees directions in the density field during early times yet no clustering. In contrast, when $\gamma/h \ll 1$, the nonlinear instability sets in much before the linear modes have a chance to significantly grow in amplitude, and the mass motion is so fast that large clusters are formed and they rapidly coalesce to form extended regions of high density, thus masking out the transient growth of the linear modes. For the case $\gamma/h \gg 1$, clustering is possible at later times when $T(t)$ increases to its asymptotic value, but mass that agglomerated is also quickly dispersed by convection. When this happens, the quantity $\sqrt{\gamma/h}$ is no longer arbitrarily large but takes an $\mathcal{O}(1)$ value which allows (25) and (26) to be satisfied simultaneously. This again implies a scale for the clustering process determined by $kl \sim \sqrt{\epsilon}$. The microstructure in this case (shown in Fig. 1) consists of moderately dense and interspersed clusters which are continually dispersed and recreated in the flow. The clusters also scatter continually into each other as they are being rotated and stretched by the mean shear, resulting in a highly time-dependent microstructure. It is noted that the stripwise organization of the clusters as shown in Fig. 1 persists despite the strong time-dependence. This organization is stabilized by a complex mechanism involving the interplay of mass agglomeration along the extensional axis of the shear, convective dispersion and cluster-cluster scattering, the details of which will be presented in [20].

To reprise, the two flow states shown in Fig. 1 and Fig. 4, which are reached by the system asymptotically after long times, correspond respectively to the case in which $T_i < T_0$ and that in which $T_i \gg T_0$. The flows shown will respectively be refered to as the 'quasihomogeneous' flow

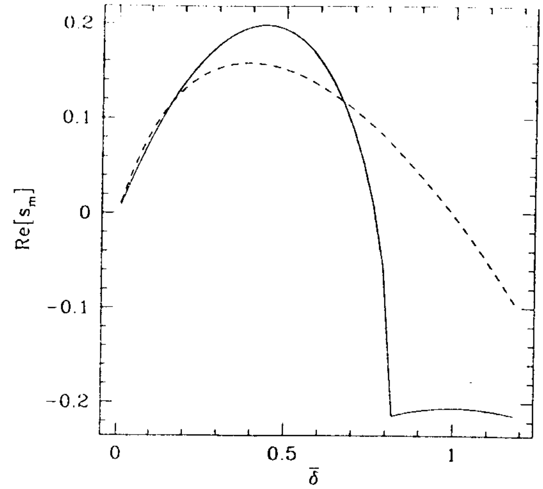

Figure 2: The real part of s_m as a function of $\bar{\delta}$ at fixed $\theta = \pi/4$ (solid line) and $\theta = 3\pi/4$ (dotted line). The value of $\sqrt{\epsilon}$ is 0.8.

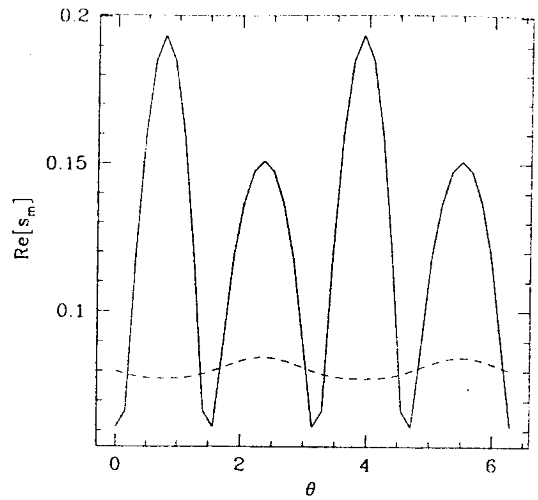

Figure 3: The real part of s_m as a function of θ for $\bar{\delta} = 0.5$ (solid line) and $\bar{\delta} = 0.1$ (dotted line). The value of $\sqrt{\epsilon}$ is 0.8.

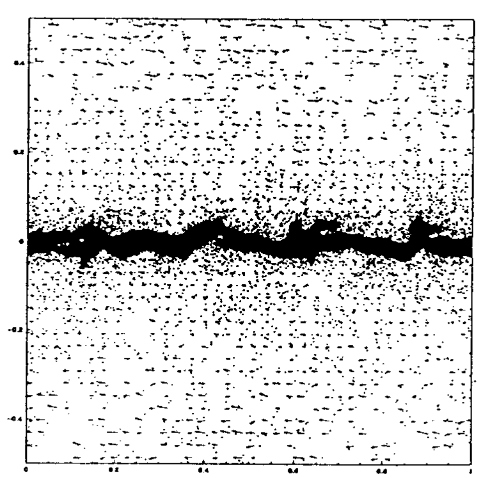

Figure 4: The particle configuration of a plugged shear flow at steady state on which a vector plot of the velocity field is superposed. The parameters characterizing the flow are the same as those of the flow shown in Fig. 1.